Name: ____________________ Partner's Name: ____________________

The Moment of Inertia

In this experiment, you will determine the Moment of Inertia of a large metallic disk system.

Today's lab will be the study of the Moment of Inertia. The Rotational Inertia apparatus is used to determine the effect of a constant torque on a rotating disk.

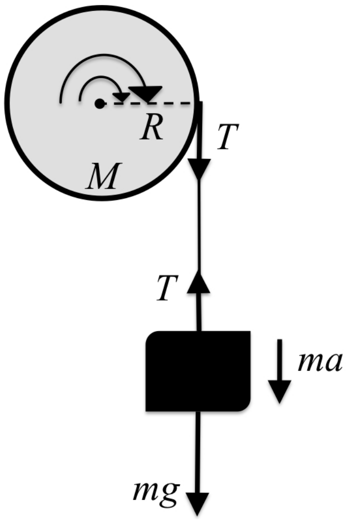

Figure 1: Moment of Inertia apparatus. A falling mass m causes the disk to rotate. The fall rate of the mass, and the rotation rate of the disk, depend upon the Moment of Inertia I of the disk.

INTRODUCTION - THE MOMENT OF INERTIA

The Moment of Inertia is that property of a body which causes it to oppose any tendency to change its state of rest or uniform angular rotation. The Moment of Inertia I of a body is the ratio of the net torque $\Sigma\tau$ acting upon it to the angular acceleration α that results,

$$I = \frac{\Sigma\tau}{\alpha}, \tag{1}$$

where the units of $[I]$=kg$\cdot$ m^2.

The rotational inertia of a geometrical object can be calculated using techniques from integral calculus. For example, the rotational inertia of a uniform cylinder of radius R and mass M rotating about its longitudinal axis is given by

$$I_{cyl} = \frac{1}{2}MR^2. \tag{2}$$

The Moment of Inertia of an object about any axis may be obtained experimentally, no matter how irregular the body, by applying a known torque τ and measuring the resulting angular acceleration α. If the torque is due to a the application of a force F with R being the lever arm, then $\tau = RF$, and Eq. (1) can be written as

$$I = \frac{RF}{\alpha}. \tag{3}$$

In this experiment, the Moment of Inertia I of a circular disk is measured by applying a know torque τ and measuring the resulting angular acceleration α. The torque on the disk is created by fastening a known mass m to a cord wrapped around the rim of the disk (see Fig. 1). To model this system, we begin with the expression for I in Eq. (3). You will measure r and α directly, but the tension force on the disk is not simply mg because the disk is accelerating.

1. Your first task is to find the tension force T in terms of m, α and g and show that Eq. (3) reduces to

$$\boxed{I = \frac{m(g - \alpha R)R}{\alpha}} \tag{4}$$

Draw a picture with all the forces clearly indicated and show your work in detail.

HINT – Use Newton's second law, $\sum F = ma$, and the relation $a = R\alpha$.

PROCEDURE:

CAUTION This apparatus uses a high voltage spark generator to mark sensitized chart paper. When the spark generator is in use, you must not touch any part of the apparatus except the insulated slider handle. If you touch any part other than the handle, you may get a painful electrical shock.

1. **Initial Setup:**
   Make sure the metal disk is on its support and the knurled screw is tightened. Wrap the thread several times around the outside rim, fastening one end to the pin and the other end to the accelerating mass. Arrange the setup so that the mass will fall one and a half to two meters.

2. **Frictional Torque:**
   Determine the frictional torque by attaching a small initial mass m_f to the string. Give the disk a small initial speed and continue to adjust m_f until the disk continues to rotate uniformly at this speed. The torque thus applied is just equal to the frictional torque $\tau_f = m_f gr$.

3. **Data Collection:**
   Attach a fresh sheet of coated paper to the wheel. Set the slider so that the spark point is near the rim of the wheel. The spark point should be adjusted so that it is slightly less than 1 mm from the chart. Attach an accelerating mass of about $m = 200$ gm in addition to m_f. Release the mass from rest and, as it descends, slide the spark point slowly towards the central axis. Adjust the speed of the slider so that the spark dots are clearly separated for the successive rotations.

   After the method of moving the spark point has been mastered connect the spark-timing device to the binding posts of the rotational inertia apparatus, turn ON the switch, release the falling mass, and gradually move the spark point toward the axis of the disk. Make sure the box is set to a frequency of 10 Hz. Just before the accelerating mass reaches the floor, turn OFF the spark timer and stop the disk. The spark timer MUST be turned OFF before the disk is touched. Examine the trace to see that there are no overlapping rings and few missing points. If a few spots are missing the trace can still be used, but if many points are missing a new record must be made.

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DATA ANALYSIS:

1. Examine the record sheet and record the values of the angle θ (in deg.) and time t in Table 1. For each complete revolution add 360^o.
2. Once all data points have been entered, subtract successive angles to find $\Delta\theta$ (i.e $\Delta\theta_2 = \theta_2 - \theta_1$) for all times.
3. Divide your $\Delta\theta$ values by the time interval between points, Δt, and record these as ω (deg/s) in Table 1.
4. Finally, complete Table 1 by converting your angular velocity values ω to the units of (rad/s).
5. Make a GRAPH of ω (rad/s) vs. time t.
6. From the graph of ω vs t, how can you find the angular acceleration α? Explain.

$$\alpha = \underline{\hspace{3cm}}$$

7. Use your value for α and Eq. (4) to determine the moment of Inertia,

$$I = \frac{m(g - \alpha R)R}{\alpha} = \tag{5}$$

$$I = \underline{\hspace{3cm}}$$

GEOMETRIC CALCULATION OF THE MOMENT OF INERTIA I:

In this section you will calculate the Moment of Inertia I of your disk system based upon it's mass and geometry. Figure 3 illustrates a schematic of the rotating disk and shafts.

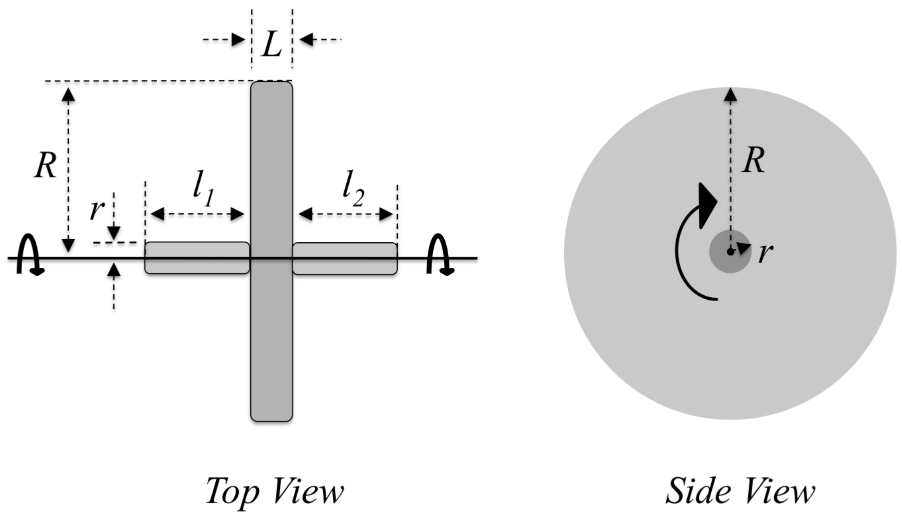

Figure 2: Top and side view of rotating disk wheel and shaft.

- $M, R, L \rightarrow$ Mass, radius, and thickness of large disk.
- $m, r, l = l_1 + l_2 \rightarrow$ Mass, radius, and (total) length of both shafts.
- $V_M = \pi R^2 L \rightarrow$ Volume of large disk.
- $V_m = \pi r^2 l \rightarrow$ Volume of both shafts.
- $V_T = \pi R^2 L + \pi r^2 l \rightarrow$ Total Volume

The first step is to measure the properties of your rotating system ...

- Total mass $M_T = M + m =$ ________________
- $R =$ ________________ $r =$ ________________ $L =$ ________________
- $l_1 =$ ________________ $l_2 =$ ________________ $l = l_1 + l_2 =$ ________________

The disk is made of two cylinders of different radii. The total Moment of Inertia I is the sum of the separate moments of inertia of the two cylinders. This can be written as

$$I = (1/2)MR^2 + (1/2)mr^2 \tag{6}$$

Now, we can't directly evaluate I in Eq. (6) because we don't know M and m separately, only their sum total M_T. To continue then, we need to rewrite the masses in terms of their relative volumes which we do know. Remember that the material density=mass/volume, with ρ being the density of the metal used, in units of kg/m^3. In terms of the total mass and volume, the density is

$$\rho = \frac{M_T}{V_T} = \frac{M_T}{(\pi R^2 L + \pi r^2 l)} \tag{7}$$

The individual masses M and m can be written in terms of ρ and their volumes V_M and V_m,

$$M = \rho \pi R^2 L \qquad \text{and} \qquad m = \rho \pi r^2 l \tag{8}$$

1. Start with the Moment of Inertia Eq. (6), and use Eqs. (7) and (8), to show that the Moment of Inertia I can be written as

$$\boxed{I = \frac{M_T}{2}\left(\frac{R^4 L + r^4 l}{R^2 L + r^2 l}\right)} \tag{9}$$

2. Use equation (9) and your measurements for M_T, R, r, L and l to solve for I.

$I =$ ________________

3. Compare your values for I obtained from the geometry here, and from rotational acceleration above. Find the % difference.

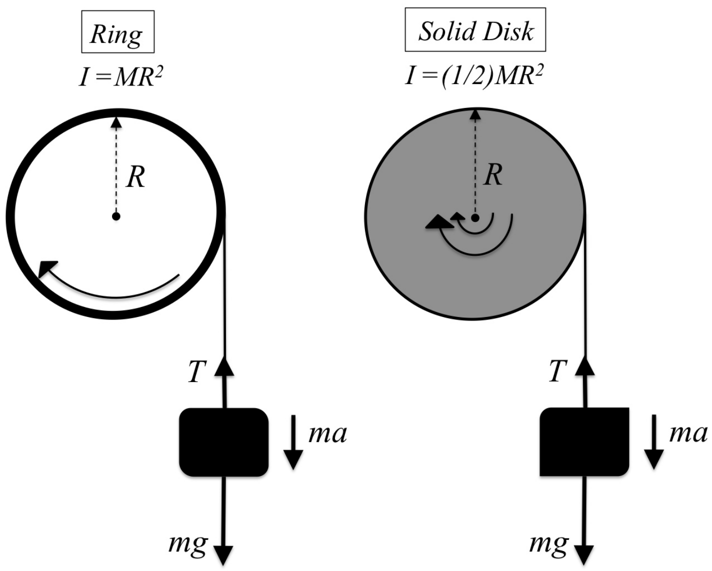

Figure 3: Side views of a ring and a solid disk, both with the same mass M and same radius R.

Figure 3 shows two rotating objects. On the left is a ring, or shell, of mass M and radius R. On the right is a solid disk, like that used here, with the same mass M and radius R as the ring. Because the *distribution of mass* is different in the two cases, their Moments of Inertia I are not the same. As a result, when the same hanging mass is used to rotate them, they will rotate, and accelerate, at different rates. You are to calculate these two rates.

4. Use Newton's second law, $\sum F = ma$, along with the torque equation, $\sum \tau = R \sum F = I\alpha$, and $a = r\alpha$ to show that the relative acceleration a/g of the mass m is given by the general relation

$$\frac{a}{g} = \frac{1}{1 + I/mR^2} \tag{10}$$

5. Now solve Eq. (10) for the acceleration of a $m = 5$ kg falling mass attached to (a) a ring or (b) a solid disk. Both the ring and the disk have radii $R = 0.1$ m and mass $M = 5$ kg. Which one falls faster? Can a ever be greater than g?

Table 1: Rotational Inertia Data

Spot #	Time (sec)	θ (deg)	$\Delta\theta$ (deg)	ω (deg/s)	ω (rad/s)
			- -	- -	- -

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DUE NEXT WEEK...

1. The lab manual pages with all measurements done and all questions answered.
2. A graph of ω vs. t with a trendline fit for α.

END LAB #9

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