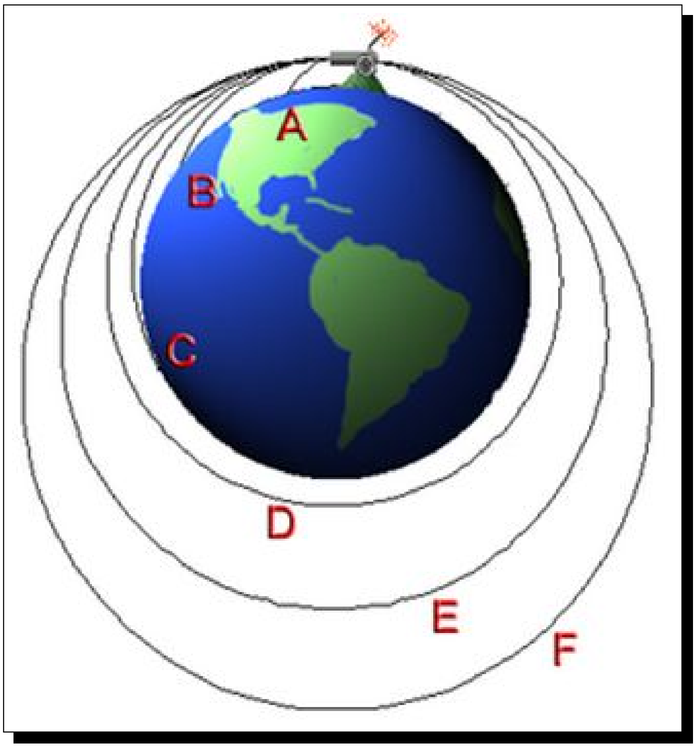
NAME: ____________________ DATE: ____________________

# Atmospheric Retention

*The NAAP Atmospheric Retention Lab explores some of the elements that go into the retention or loss of an atmosphere by a planet. The Maxwell-Boltzmann velocity distribution and escape velocity are introduced.*



## 1. Gravity and Orbits

There is the old adage that what goes up, must come down. When thinking about gravity, Isaac Newton developed the concept of an orbit. He imagined a cannon on top of a tall mountain. When the cannon ball is shot, the cannon ball travels a certain distance (path A). If more gunpowder is used, the cannon ball is shot with greater speed and travels even farther (paths B and C). Knowing that the earth was round, Newton thought about the path the ball would take if it were shot with even greater speed. In his simplistic thought experiment, the ball could actually curve around the earth and hit the back of the cannon (path D). Such a fast moving ball would be said to be in a circular orbit. Such an object will never come down - they continually "miss" the earth since the Earth's gravity is continually redirecting the ball. Nevertheless, they are still bound to the earth and cannot escape. Firing the cannon ball with even greater speeds results in elliptical orbits (paths E and F).

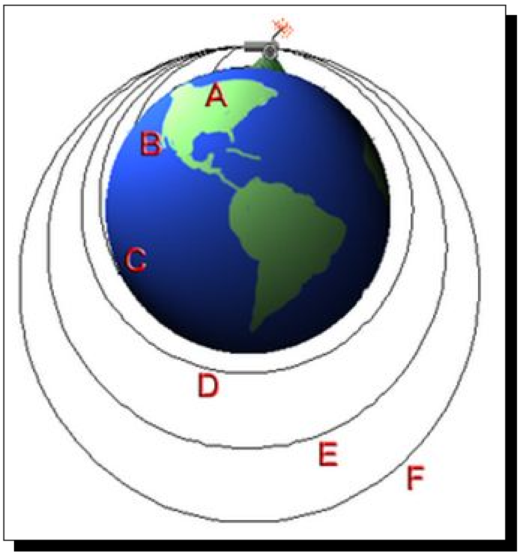



**Figure 1:** Increasing velocity may result in an orbit. Note that orbits need not be circular.

### *1.1. Escape Velocity*

Now instead of shooting the cannon sideways, imagine a similar experiment where the cannon is shot straight upwards. Each time more gunpowder is added, the cannonball travels to a greater height and takes longer to come back down. In the limit of our thought experiment the ball can go so high that it would take forever to come back down. Such a ball is said to have escaped as it will never return. The speed necessary to launch an object up such that it will never come back is well defined for masses because the basics of how gravity works is well understood. This speed is called the *escape velocity* and it is defined as:

$$v_{escape} = \sqrt{\frac{2GM}{R}} \tag{1}$$

Note that the escape velocity depends upon both the mass M and radius R of a body and uses Newton's Gravitational Constant $G = 6.67 \times 10^{-11}$ Nm$^2$/kg$^2$. Its value for the Earth is 11,200 m/s. A particle will escape if it has a one-time vertical component of velocity greater than this value. Note that this is very different from the method by which rockets escape from the Earth by continually providing thrust. Rockets escape even though their speeds are much less than the escape speed.

## 2. Projectile Simulation

The **Projectile Simulator** you are to use in this lab is located at

http://astro.unl.edu/naap/atmosphere/projectile.html

It allows one to determine the escape velocities for various bodies by observing the behavior of projectile shot vertically from their surfaces. There are many simplifications used in this simulator such as ignoring atmospheres, rotation, and other gravitational influences and the bodies are assumed to be spherical and of uniform density. The simulator will be set for the Earth in default mode, but you can simulate other bodies by changing the Mass and Radius. There is also an animation rate setting allowing you to control how time passage in the simulator maps to actual time passage.

Use the Projectile Motion Simulator to experiment with firing a high-powered rifle bullet vertically from the surface of the Earth. Would you expect such a bullet to escape from the Earth? Run the simulator for a muzzle velocity of 1400 m/s. Note how the velocity of the bullet decreases due to the gravitational influence of the Earth until it reaches its peak height. Then the velocity becomes negative and increases in magnitude as the projectile returns to Earth.

Now change the projectile velocity to 15 km/s and fire again (with a high animation rate). Note that after a day passes the velocity really doesn't decrease any more thus the projectile was launched with a speed greater than the escape velocity. Since a projectile fired with exactly the escape velocity will come to a velocity of zero only after an infinite amount of time, it isn't practical to try and precisely determine these values with this simulator. However, we can estimate their values by noting the velocity range in between (1) where the particle clearly returns to Earth and (2) where the particle's velocity clearly does not decrease with altitude and the particle clearly escapes.

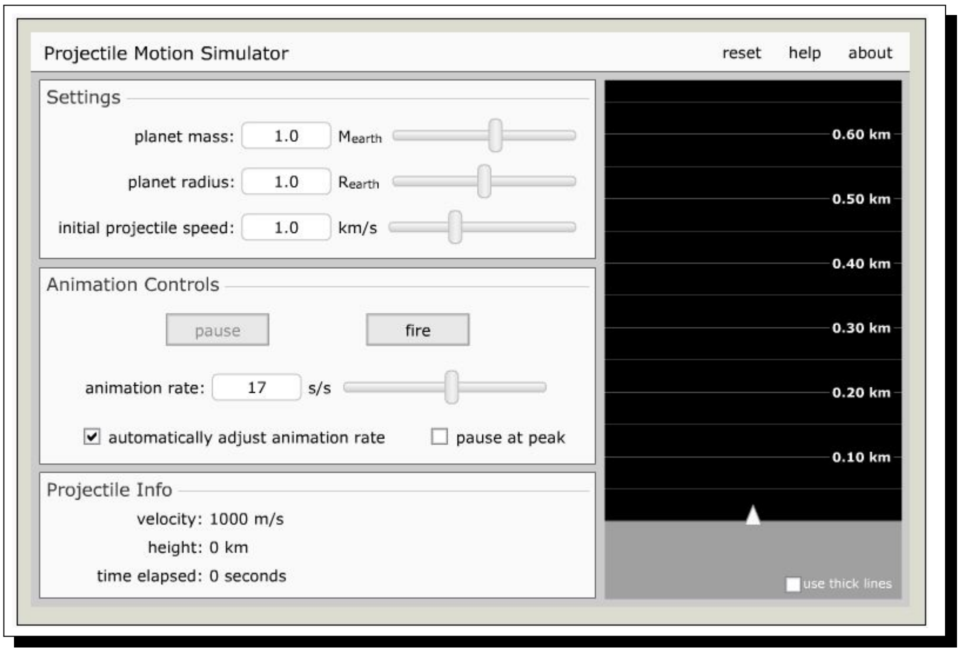



**Figure 2:** Projectile motion simulator at http://astro.unl.edu/naap/atmosphere/projectile.html.

### 3. Speed Distribution - Maxwell Distribution

A sample of a gas is made of a large number of particles. For example, 4.0 grams (the atomic mass) of Helium used in a blimp has $6.02 \times 10^{23}$ (Avogadro's Number) individual atoms of helium which is known as a mole. *Gas particles are not all moving with the same speed, they have a distribution of speeds.* Any one particle could be moving very fast or very slow. This distribution of speeds is greatly affected by the temperature of the gas. Molecules in a hot gas are on average moving "faster" than molecules in a cold gas.

The **Maxwell velocity distribution** for molecule speeds in a gas can be found at

http://astro.unl.edu/naap/atmosphere/distribution.html

While it is not necessary for the lab to know the mathematical equation, it is important to note a few general features of the Maxwell distribution of particle speeds in a gas:

1. The distribution has a crude "bell-shape" which peaks at $v_{mp} = \sqrt{2kT/m}$, the **most probable speed**.
2. The exact shape of the distribution is not symmetric. There is a high-speed tail to the distribution.
3. The average velocity (speed and direction) is zero. The magnitude of the average speed $v_{avg}$ is not found at the peak, but a little past it because of the high-speed tail.

The simulation of the distribution below allows you to animate certain velocity ranges of the particles. Each tracer dot represents a great many particles. Note that the distribution is a function of the particle mass, thus it is different for each type of gas. The Hydrogen curve will be broader and wider than the distribution of a more massive gas like Carbon Dioxide. The mass of a gas particle can be conveniently expressed in terms of atomic mass units $u = 1.66 \times 10^{-27}$. A particle has roughly 1 u for each proton or neutron in an atom. Thus, a methane ($CH_4$) molecule would have a mass of 16u (12 for carbon which has 6 protons and 6 neutrons and 4 for hydrogen which only has a proton). One mole of methane would have a mass of approximately 16g and contain $6.02 \times 10^{23}$ molecules.

Statistical Mechanics is the branch of physics concerned with relating the microscopic behavior of the particles of the gas to the macroscopic quantities that we measure. The result from statistical mechanics states that the average kinetic (motion) energy of a particle is proportional to the temperature which allows one to solve for the **average velocity** which is

$$v_{av} = \sqrt{\frac{3kT}{m}} \ , \tag{2}$$

where $m$ is the mass of an individual atom, and $k = 1.38 \times 10^{-23}$ m$^2$kg/ s$^2$K is the Boltzmann Constant.

### Atmospheric Loss

Imagine a volume of gas at the top of a planets atmosphere. The gas will have a range of velocities described by the Maxwell distribution. If a particle is moving sufficiently fast (i.e. with a speed greater than the escape velocity) and moving away from the planet, it can escape into space. If the escape velocity is low enough, the gas will be depleted fairly quickly. However, typically the escape speed is very far into the high-speed tail of the Maxwell distribution so only a very few particles will be able to escape. But once these particles are lost, the high-speed tail will be replenished and then lost again. Thus, even if the escape speed is well into the high-speed tail, it can slowly "bleed off" the atmosphere. The constitution of planetary atmospheres has been determined by the interplay between escape velocity and the Maxwell distribution over the 4.6 billion year history of our solar system.

## 4. Background Information

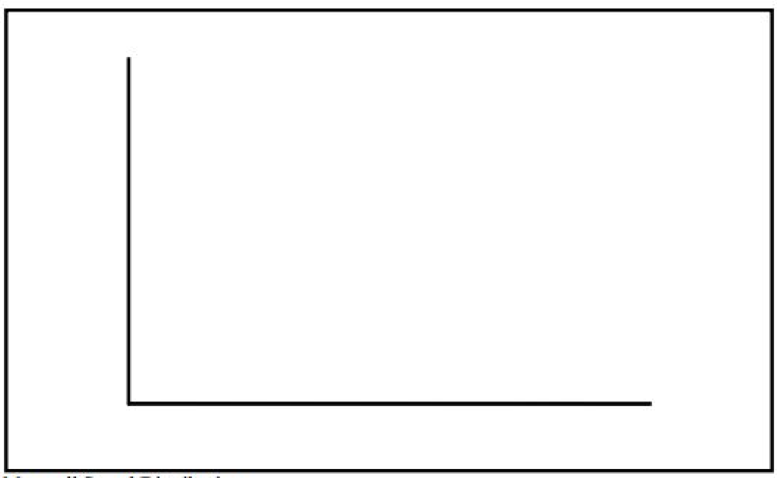
Work through the background sections on Escape Velocity, Projectile Simulation, and Speed Distribution. Then complete the following questions related to the background information.

**Question 1:** Imagine that asteroid A that has an escape velocity of 50 m/s. If asteroid B has twice the mass and twice the radius, it would have an escape velocity that is how many times the escape velocity of asteroid A?

**Question 2:** Complete the table below by using the **Projectile Simulator** to determine the escape velocities for the following objects. Since the masses and radii are given in terms of the Earth's, you can easily check your values by using the mathematical formula for escape velocity. (Remember, for the Earth, $R_E = 6.36 \times 10^6$ m/s, and $M_E = 5.97 \times 10^{24}$ kg.)

| Object | Mass ($M_E$) | Radius ($R_E$) | $v_{esc}$ (km/s) | $v_{esc}$ theory $\sqrt{2GM/R}$ |
|---|---|---|---|---|
| Earth | 1.00 | 1.00 | 11.2 | 11.2 |
| Mercury | 0.055 | 0.38 | | |
| Uranus | 15 | 4.0 | | |
| Io | 0.015 | 0.30 | | |
| Vesta | 0.00005 | 0.083 | | |
| Krypton | 100 | 10 | | |

**Question 3:** Experiment with the Maxwell Distribution Simulator. Then (a) draw a sketch of a typical gas curve below, (b) label both the x-axis and y-axis appropriately, (c) draw in the estimated locations of the most probable velocity $v_{mp}$ and average velocity $v_{avg}$, and (d) shade in the region corresponding to the fastest moving 3% of the gas particles.


Maxwell Speed Distribution

**5. Gas Retention Simulator**

Open the **gas retention simulator** at

*http://astro.unl.edu/naap/atmosphere/animations/gasRetentionSimulator.html*

and familiarize yourself with the capabilities of the gas retention simulator through experimentation.

- The **gas retention simulator** provides you with a chamber in which you can place various gases and control the temperature. The dots moving inside this chamber should be thought of as tracers where each represents a large number of gas particles. The walls of the chamber can be configured to be a) impermeable so that they always rebound the gas particles, and b) sufficiently penetrable so that particles that hit the wall with velocity over some threshold can escape. You can also view the distributions of speeds for each gas in relation to the escape velocity in the **Distribution Plot** panel.
- The lower right panel entitled gases allows you to add and remove gases in the experimental chamber. The lower left panel is entitled chamber properties. In its default mode it has allow escape from chamber unchecked and has a temperature of 300 K. Click **start simulation** to set the particles in motion in the chamber panel. Note that **stop simulation** must be clicked to change the temperature or the gases in the simulation.
- The upper right panel entitled distribution plot allows one to view the Maxwell distribution of the gas as was possible in the background pages. Usage of the show draggable cursor is straightforward and allows one to conveniently read off distribution values such as the most probable velocity. The show distribution info for selected gases requires that a gas be selected in the gas panel. This functionality anticipates a time when more than one gas will be added to the chamber.

**EXERCISES**

- Use the pull-down menu to add hydrogen to the chamber.

**Question 4:** Complete the table using the draggable cursor to measure the most probable velocity for hydrogen at each of the given temperatures. Write a short description of the relationship between $T$ and $v_{mp}$.

| T (K) | $v_{mp}$ (m/s) |
|---|---|
| 300 | |
| 200 | |
| 100 | |

**Question 5:** If the simulator allowed the temperature to be reduced to 0 K, what would you guess would be the most probable velocity at this temperature? Why?

- Return the temperature to 300 K. Use the gas panel to add Ammonia and Carbon Dioxide to the chamber.

**Question 6:** Complete the table using the draggable cursor to measure the most probable velocity at a temperature of 300 K and recording the atomic mass for each gas. Write a short description of the relationship between mass and $v_{mp}$ and the width of the Maxwell distribution.

| Gas | Mass (u) | $v_{mp}$ (m/s) |
|---|---|---|
| $H_2$ | | |
| $NH_3$ | | |
| $C0_2$ | | |

**Question 7:** Check the box entitled **allow escape from chamber** in the chamber properties panel. You should still have an evenly balanced mixture of hydrogen, ammonia, and carbon dioxide. Run each of the simulations specified in the table below for the mixture. Click **reset proportions** to restore the original gas levels. Write a description of the results similar to the example completed for you.

| Run | T (K) | $v_{esc}$ (m/s) | Description of Simulation |
|---|---|---|---|
| 1 | 500 | 1500 | $H_2$ is very quickly lost since it only has a mass of 2u and its most probable velocity is greater than the escape velocity, $NH_3$ is slowly lost since it is a medium mass gas (18u) and a significant fraction of its velocity distribution is greater than 1500 m/s, $CO_2$ is unaffected since its most probable velocity is far less than the escape velocity. |
| 2 | 500 | 1000 | |
| 3 | 500 | 500 | |
| 4 | 100 | 1500 | |
| 5 | 100 | 1000 | |
| 6 | 100 | 500 | |

**Question 8:** Write a summary of the results contained in the table above. Under what circumstances was a gas likely to be retained? Under what circumstances is a gas likely to escape the chamber?

**6. Gas Retention Plot**

This gas retention plot, located at

*http://astro.unl.edu/naap/atmosphere/animations/gasRetentionPlot.html*

presents an interactive plot summarizing the interplay between escape velocities of large bodies in our solar system and the Maxwell distribution for common gases. The plot has velocity on the y-axis and temperature on the x-axis. Two types of plotting are possible:

- A point on the graph represents a large body with that particular escape velocity and outer atmosphere temperature. An active (red) point can be dragged or controlled with sliders. Realize that the escape velocity of a body depends on both the density (or mass) and the radius of an object.
- A line on the graph represents 10 times the average velocity ($10 \times v_{avg}$) for a particular gas and its variation with temperature. This region is shaded with a unique color for each gas.
- If a body has an escape velocity vesc over $10 \times v_{avg}$ of a gas, it will certainly retain that gas over time intervals on the order of the age of our solar system.
- If $v_{esc}$ is roughly 5 to 9 times $v_{avg}$, the gas will be partially retained and the color fades into white over this parameter range.
- If $v_{esc} < 5 \times v_{avg}$, the gas will escape into space quickly.

**EXERCISES**

• Begin experimenting with all boxes unchecked in both the gasses and plot options.

**Question 9:** Plot the retention curves for the gases hydrogen, helium, ammonia, nitrogen, carbon dioxide, and xenon. Explain the appearance of these curves on the retention plot.

**Question 11:** Drag the active point to the location (comparable with the escape speed and temperature) of Mercury. The gases hydrogen, helium, methane, ammonia, nitrogen, and carbon dioxide were common in the early solar system. Which of these gases would Mercury be able to retain?

END LAB #7